The definition of a magnetic monopole in Electrodynamics combined with Gravitation

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Abstract

It is discussed the singular string associated to the gauge field of monopoles must be a physical observable if the monopole charge g is different from zero, $g \neq 0$. It is naturally found that if the gauge is to be an observable, it is possibly connected to gravity.

1   Introduction

Some non solved fundamental theoretical questions are involved in the definition of the magnetic charge or monopole in Quantum Electrodynamics. One of these questions is the non - observable singular gauge string (many times called Dirac string). Electric charges must never touch such strings according to the original Dirac's proposal, [1,2]. In section 2 it is discussed the general problems related to Dirac monopoles [3,4] and the topologycal solution of Wu and Yang [5,6,7] where the gauge theory is identified to a Fiber - Bundle geometric structure. According to this view Dirac strings are said to be non - observable. In the same section we cite the important recent work of He, Qiu and Tze [8] in which it is proven no magnetic charge different from zero in Quantum Electrodynamics is possible if the string is not a physical observable. This cleary shows that pure Quantum Electrodynamics (as we know it) does not supports monopoles in any way.

We follow identifying that in general the monopole string may have some volume in the three - dimensional flat world. This implies that for $g \neq 0$ (the

monopole's charge different from zero) one has to consider a reduced physical world for the particles and fields in interaction with the monopole. The string is the place where the gauge potential is not defined, and as a consequence (in the general case of volumetric string) the monopole's magnetic field is also not defined in this region. In section 3 we consider Gravity as seen in a three - dimensional flat world, showing there are forbidden places for particles and fields in the case even a small spherical mass is present. Defining the electromagnetic vector potential as having a volumetric singular string, it is the same as to define it in a reduced physical space relative to the three - dimensional flat space. As the physical spacetime in the presence of some mass can be seen as this smaller or reduced space (subspace in some formulations, [6]), the monopole is defined in the combined theory of Electromagnetism and Gravity. This means monopole gauge strings are observable as spacetime distortions. Other interpretations and studies on this connection are addressed to a future work in the concluding subsection of section 3.

2   The definition of a monopole

Magnetic charge or monopole is a open theoretical and experimental question. The original version of the monopole proposed by Dirac [1] never found a place in Nature as we know it: It has never been found by any tangible experiment and the theoretical machinery used to define its properties never fulfil all requirements in terms of what we understand by a particle in Electrodynamics.

The string of singularities has been a problem of non - natural assumptions in Quantum Electrodynamics since Dirac's proposal [1]. In order to avoid the string Dirac defined a *nodal line*, a region of the spacetime where all Schroedinger wave functions associated to the particles in the Universe are identically zero. In the Schroedinger equation, $(-i\nabla - e\mathbf{A})^2\psi = 2m\mathcal{E}\psi$, it is necessary for ψ to be zero or discontinous over the string of singularities. The nodal line is then a constraint on the places the particles can be found - no particle is allowed to be in some determined (by the strings positions) places of the Universe.

After some time of theoretical existence, the monopole idea was improved by Dirac himself in terms of a variational principle formulation and the problems related to it, [2]. The source of problems again came related to the

definition of the singularity line of the vector potential. It is necessary to assume that a string of such type never pass through a charged particle if the equations of motion are to be derived from an action principle. In trying a quantized version of the theory via a Hamiltonian formulation, Dirac [2] found that the Poisson Brackets for the vector - potential field is directly dependent on an arbitrary function which, in its turn, is related to the mechanism used to define a possible action principle (i.e., without the divergences that arise from the singular strings). Up to that point the string was the theoretical difficulty which required something the basic principles in Electrodynamics could not give: A topologycal explanation in which the string could find a place as a gauge artifact.

Almost twenty years later, Rohrlich [3] and Rosenbaum [4] showed in a different approach the conditions for the existence of a variational principle valid in the case of monopoles. It is interesting to quote the problem put forward by Rohrlich that a system of particle equations can be derived only from a nonlocal action integral and that no action integral exists from which both the particle and field equations can be derived. As in Rosenbaum work, the idea is to have a non - natural contraint (not derived from the action principle) about the dynamics of charges and monopoles: Charges must never touch monopoles; Since Lorentz force can be derived from a principle which states that a charge approaching a monopole along a straight line will collide, it is in contradiction with the necessity of a charge never pass through a monopole. These considerations indicated that something strange to Electrodynamics should be assumed in order to not fall into contradiction about the theoretical existence of the monopole. In fact, some years later t 'Hooft and Polyakov [5] proposed that a natural condition for the monopole to exist will arise when other topologies are involved, i.e., when other forces (associated to broken non - Abelian gauge theories) of Nature are in consideration.

About the same time Wu and Yang [6] gave a clever topologycal description of the problem in non - Abelian as well as in the Abelian case. Using the concept of Fiber Bundles their proposal is that along a path of some charged particle (in the $U(1)$ case), the gauge can be changed in order to avoid the string of singularities. Each gauge changing is well defined if Dirac Quantization Condition is respected - which is quite obvious since a phase changing of 2π will not alter Physics. In this view no singularities are seen by any particle's path and a beautifull connection to geometric concept of fields is naturally given.

Soon after Wu and Yang's work, Brandt and Primack [7] showed that

original Dirac's theoretical formulation [1] of the monopole is equivalent to the Wu and Yang's one [6]. Showing that the Dirac string attached to the monopole can be arbitrarily moved by a gauge transformation valid everywhere and that it can be completely removed at the cost of introducing a non - global topology in space, Brandt and Primack accomplished to demonstrate the equivalence of the formulations.

Recently He, Qiu and Tze [8] proposed the inconsistency of Quantum Electrodynamics in the presence of monopoles. These authors [8] formally show that the gauge coupling associated with the unphysical longitudinal photon field is non - observable and has an arbitrary value in Quantum Electrodynamics. In deriving Dirac Quantization Condition in $U(1)$ case it was found that this derivation involves only the unphysical longitudinal coupling constant. This work was focused on Dirac point - monopoles in the standart Dirac formulation [1,2]. It is interesting to observe that according to Brandt - Primack [7] and He - Qiu - Tze [8] works, Quantum Electrodynamics is inconsistent in the presence of monopoles in whichever formulation. The singular Dirac string in the monopole gauge potential is a purely gauge artifact, it is just a gauge freedom which allows one to arbitrarily move the string around without any physical effect, provided Dirac Quantization Condition is satisfied. He, Qiu and Tze [8] observed that by introducing another unphysical pure gauge field into Quantum Electrodynamics, it is possible to attribute part of the singularities to this pure - gauge field, and thus the corresponding Quantization Condition involves the unphysical gauge coupling associated with this pure - gauge field. After Dirac monopoles are introduced in whichever way, the exact $U(1)$ gauge invariance must be respected, so that fixing any specific physical value for an unphysical gauge coupling will violate the exact $U(1)$ gauge invariance. Conversely, *if we assume $g \neq 0$ the string must be observable* according to reference [8]. It is impossible to have $g \neq 0$ and require no physical effects due to the singular string as Dirac [1,2] or Wu and Yang [6] claimed, [8]. This implies that some new fundamental principle should exist, which gives this pure - gauge coupling (and consequently to the longitudinal photon field) a physical meaning so that it become observable [8]. Accordingly, He, Qiu and Tze thought this is most unlikely and set $g = 0$ as the only possible solution in pure Quantum Electrodynamics.

Aharonov and Bohm proposed an experimental test for the quantization of the magnetic flux related to the discret value of the electron's charge, [9]. A magnetized iron filament called whisker is positioned inside the volume of the environment where electrons are allowed to perform trajectories, i.e.,

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these electric charges are not allowed to go inside the whisker region. This simple case has no problem to be defined theoretically: The reason is because there is a physical limitation (some material in the whisker region) for the electric charges to not cross over the magnetized filament.

It turns out to be impossible to define a singularity - free potential A^μ over all the flat three - dimensional world (i.e., over all $\mathbf{R^3}$,[6]). In fact as in reference [6] any fiber of the physical environment is smaller than $\mathbf{R^3}$, the physical space cannot be defined as the entire $\mathbf{R^3}$ flat world for a given gauge.

The problem related to the monopole's string can be understood in terms of the singular region associated to the electromagnetic potential A^μ, without which no quantization of the angular momentum eg/c is defined (e is the quantum of electric charge and g is the quantum of magnetic charge). The main problem is that this singular part of the A^μ potential implies an infinite amount of interaction energy (self or mutual). A simple example of a vector potential is:

$$\mathbf{A} = -g\frac{1 + \cos\theta}{r\sin\theta\, step\,[\theta - \delta]}\phi \qquad (1)$$

where ϕ is the unitary vector associated to the spherical coordinate ϕ, and θ is the other spherical coordinate in $\mathbf{r} = (\mathbf{r}, \theta, \phi)$ defined in a *three - dimensional flat spacetime* ($\mathbf{R^3}$ according to the terminology of this work). The "step" function is defined as $step[x-x_0] = 0(x < x_0); 1/2(x = x_0); 1(x > x_0)$, and δ is a vanishing angle, $\delta \to 0$. The magnetic field derived from this potential ($\mathbf{B} = \nabla \times \mathbf{A}$) is:

$$\mathbf{B} = g2\pi(1 + \cos\delta)\frac{\mathbf{r}}{r^3} \qquad (2)$$

if $\delta < \theta \leq \pi$, any ϕ, otherwise the magnetic field is not defined. The energy proportional to $\int \mathbf{B_i} \cdot \mathbf{B_j} \mathbf{d^3x}$ (the indices refer to different sources $i \neq j$, or the same source $i = j$) is not defined for $\theta \leq \delta$, any ϕ (the string region), as well as the momentum proportional to $\int \mathbf{E_i} \times \mathbf{B_j} \mathbf{d^3x}$.

When $\delta = 0$ it is said that $\mathbf{A}$ remains singular but $\mathbf{B}$ is well defined everywhere. The general situation happens when the singular string has some volume however ($\delta \neq 0$ in the example).

The theoretical question about Dirac monopoles can be described in two items: i) The general formulation in terms of an action principle which allows an Hamiltonian formulation and consequent quantization, and ii) The problem involved in defining a hidden (non observable) string of singularities.

The main pourpose of the present work is to deal with the string problem (*ii*) and address to the first (*i*) in another work.

Lets consider the magnetic field is given by the rotational of the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$, in the example as given in equations (1) and (2). If it is said $g \neq 0$ then it must be assumed the physical space where the magnetic flux is measured, $\oint \mathbf{B} \cdot \mathbf{d}\sigma$ ($\mathbf{d}\sigma$ the elementary oriented area surface), is smaller than the $\mathbf{R^3}$. *In saying this flux has a non - zero net value, one is assuming the space is no longer the entire three - dimensional flat because nor particles or fields are allowed to be in string region.* The direction of the singular string in $\mathbf{R^3}$ world has no physical meaning since now it is considered no hole exists in the *reduced space* (the actual physical space where $g \neq 0$).

The strings have some three - dimensional volume in general. The general case for bosons and fermions particles studied by Weisskopf [10] put limitations on this particles radii even in the general quantum - relativistic formulation. Of course the radius of some particle is related to the not - well defined interaction between two points which are the same in spacetime. This issue is a specific part of the definition of particles which is not the focus in the present work, and it will be considered in general that a string which is to be attached to a particle must have some nonzero volume accordingly, [10]. Observe that the problem Dirac [1,2] faced with the strings is of the same kind Weisskopf did, [10]: The fields must (in Dirac's words, [2]) *go out of existence* at some places in order to define particles. The news in the monopole case is that this is an extended region in space.

In the next section we intend to provide a definition of $g \neq 0$ in the combined situation of having Electromagnetism and Gravitation. This conjunction provides a natural place for the singular string of the gauge fields. This formulation is different from that of Dirac [1,2] or Wu - Yang [6] because the effect of the string is a physical observable.

3   The observable singular string

In the last section it was discussed that the existence of a monopole in $U(1)$ case ($g \neq 0$ in Quantum Electrodynamics) implies in general the space ones takes as physical is in fact smaller than that known as the flat $\mathbf{R^3}$. The direction of the undefined region in $\mathbf{R^3}$ has no physical meaning once one accepts $g \neq 0$. It is very interesting and important to observe that this comes from the fact $\mathbf{A}$ *is not defined* in the string region - the value $1/0$ (one over

zero) has no physical or mathematical meaning in describing fields.

In order to consider $g \neq 0$ it is necessary to have a physical reason to the fields and particles never be defined in the string region. We found gravity is a possible candidate to hide the string.

3.1 A physical place for the singular string

Now it is considered the effects of Einstein's gravity in terms of deformations in spacetime as viewed in the flat three - dimensional world. This is a known valid and feasible way to see gravity, [11].

The curved spacetime interval outside the region of some matter distribution can be written as:

$$ds^2 = A(r)dt^2 - B(r)dr^2 - C(r)r^2d\theta^2 - C(r)r^2\sin^2\theta d\phi^2 \tag{3}$$

where r, θ, ϕ are regarded as spherical coordinates and $A(r)$, $B(r)$, $C(r)$ are given functions of r. It is possible to show that for $r = R$ the corresponding physical area Δ for R fixed is:

$$\Delta = 4\pi R^2 C(R) \tag{4}$$

and that the physical distance Λ between the points $r = R_0$ and $r = R$ on a given radial line is:

$$\Lambda = \int_{R_0}^{R} \sqrt{B(r)}dr. \tag{5}$$

Consider a spherical mass m at the center of the coordinate system with some given radial matter distribution function. In this particular case the interval for $r > 2m$ is:

$$ds^2 = (1 - \frac{2m}{r})dt^2 - (1 - \frac{2m}{r})^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \tag{6}$$

according to Schwarzschild. In this particular case the function $C(r)$ is equal to one. We are interested in to study this metric as seen in the $\mathbf{R^3}$ world (three - dimensional flat). For the parameter $r = R$, the physical area Δ is $4\pi R^2$. Let us determine the physical radius Λ:

$$\Lambda = \int_{R_0}^{R} r/\sqrt{r^2 - 2mr}dr = \left[\sqrt{(r^2 - 2mr)} + m\ln\left(r - m + \sqrt{(r^2 - 2mr)}\right)\right]_{R_0}^{R}, \tag{7}$$

where R_0 is some internal radius ($R_0 < R$) in the region of the matter distribution. We assume that the resulting value of the expression above for the constant parameter R_0 is vanishingly small (of the order of $2m$) compared to the one for R, so that the physical radius is:

$$\Lambda = \sqrt{(R^2 - 2mR)} + m \ln \left(R - m + \sqrt{(R^2 - 2mR)} \right) \qquad (8)$$

and for $R >> 2m$,

$$\Lambda = R + m \ln (2R - 2m) \qquad (9)$$

i.e., the physical distance Λ is larger than the parameter R.

The conclusion is that for some physical distance Λ the area avaliable to cover a sphere of this radius is $4\pi R^2$, with $R < \Lambda$, in the $\mathbf{R^3}$ world. In the three - dimensional flat world it is impossible to close the surface of radius Λ in this case. For a particle in the physical avaliable world no hole occurs, it is all continuous, but in the $\mathbf{R^3}$ world there is a region where the spacetime is not defined. As each spherical surface has this non - physical region, it performs a volume of forbidden places for all particles and fields in $\mathbf{R^3}$.

It is known that for a flat three - dimensional world it is possible to define an average curvature by means of a defect from $4\pi r^2$ of the measured area of some surface of radius r. The connection of this idea to the theory of gravitation is via a conceptual significance of the G_4^4 component of the stress - energy tensor. It is the average curvature $R_{12}^{12} + R_{23}^{23} + R_{13}^{13}$ of the three - space, which is perpendicular to the time. This is a known valid interpretation of the theory of gravitation, [11].

Now we have two physical results in $\mathbf{R^3}$: The electromagnetic potential $\mathbf{A}$ according to Maxwell's Electromagnetic theory and the physical region for particles and fields according to Einstein's gravitational theory. The electromagnetic potential is physical only outside the singular (in general volumetric) string region. The spacetime is only physical outside some region defined by the holes on each spherical surface starting from the monopole of mass m, as viewed in the flat $\mathbf{R^3}$.

The two forbidden regions in the $\mathbf{R^3}$ world can be set to be the same. It is only necessary to define a volumetric string as source for the electromagnetic potential (where $\mathbf{A}$ is not defined) exactly in the region the physical space is also not defined in the flat world. The measure $\oint \mathbf{A} \cdot \mathbf{dl}$ can be set to be a constant for any closed loop around the string (in the physical region

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already) if we require a constant flux at the string for any distance from the monopole.

This only means that in order to define a monopole the spacetime must be modified accordingly. The same situation happened to Weisskopf [10], who defined a forbidden spherical region in spacetime where *all* the fields are non - existent. In the monopole case this forbidden region is extended in spacetime.

3.2 A fiber - bundle formulation

It is possible to describe the resulting physical situation of a reduced space due to gravitation in a fiber - bundle formalism when we identify the gauge given by $\mathbf{A}$ in $\mathbf{R^3}$ as determining the region of the singular string (or the forbidden region in $\mathbf{R^3}$). As in Wu - Yang formalism [6], each gauge determines a fiber or subspace which is smaller than the flat three - dimensional world.

The view in Wu and Yang's work is that by the gauge freedom it is possible for a particle to be defined in all $\mathbf{R^3}$ since it is possible to rotate some subspace (which is smaller than the entire $\mathbf{R^3}$) to turn it into another, making some forbidden region avaliable.

In interpreting the effect of gravity as a reduction of the physical space regarded to the flat three - dimensional world, there exists a forbidden region for particles in this world. This region can be arbitrarily set on some direction in the referred space with no physical consequences whatsoever. We defined a gauge where the singular string is at the same region in $\mathbf{R^3}$ where particles are forbidden to go due to gravity, so that a gauge is associated to the position of the string in flat three - dimensional space.

In any of the two descriptions (the present one or Wu - Yang's) each gauge is associated to a reduced space relative to the flat three - dimensional one. The difference between the two formalisms comes from the fact that in the present case the interpretation is so that the reduced space for some gauge means in fact the physical avaliable space is reduced, i.e., the spacetime is distorted (and strings are observable); Changings on the string position will not result all $\mathbf{R^3}$ can be visited.

3.3 The fundamental question and conclusions

The fundamental question about a non - physically observable gauge string is that g must be exactly null if gauge invariance is to be respected in Quantum

Electrodynamics. This is one of the main causes all theoretical formulations about Dirac monopoles are inconsistent [8].

The present work proposes a definition of the monopole in a combined situation where Gravity and Electrodynamics are considered at the same time. It is discussed the electric charges and fields never touch a monopole string due to Gravity, i.e., Gravity becomes the physical observable if $g \neq 0$. It is in agreement with the conclusions in reference [8] where it is shown $g \neq 0$ implies the string must be a physical observable.

He, Qiu and Tze, [8], discussed also that in the case of a point monopole the physical observable is connected to the longitudinal photon field. In ordinary Quantum Electrodynamics this field is not physical and consequently g must be zero in pure QED (because the longitudynal photon field is not a physical observable). In view of the present result we can confirm that string is physical when $g \neq 0$ in the general case when it has some volume in $\mathbf{R^3}$, but it is not clear if in this case the longitudinal photon field has some direct role. This deserves more investigation in a future work.

4   Acknowledgements

5   References

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